High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A small particle of mass m moves in such a way that the potential energy U = $\frac{1}{2}mb^2r^2$, where b is a constant and r is the distance of the particle from the origin (Nucleus). Assuming Bohr model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to $\sqrt{n}$.

2. Suppose the potential energy between electron & proton at a distance r is given by $\frac{-ke^2}{3r^3}$. Use Bohr's theory to obtain energy levels of such a hypothetical hydrogen atom.

3. In a transition to a state of excitation energy 10.19 eV a hydrogen atom emits a 4890 A° photon. Determine the binding energy of the initial state. Also find the nature of transition?

4. Suppose in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number n change by 2 (i) Find the smallest wavelength emitted by hydrogen (ii) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm)

5. Find the velocity of photoelectrons liberated by electromagnetic radiation of wavelength λ = 18.0 nm from stationary He^+ ions in the ground state.

6. (I) Find the maximum wavelength λ of light which can ionize a H-atom in ground state.
(II) Light of wavelength λ is incident on a H-atom which is in its first excited state. Find the kinetic energy of the electron coming out.

7. A beam of monochromatic light of wavelength λ ejects photoelectrons from a cesium surface (Φ = 1.9 eV) These photoelectrons are made to collide with hydrogen atoms in ground state .Find the maximum value of λ for which (a) hydrogen atoms may be ionised (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light.

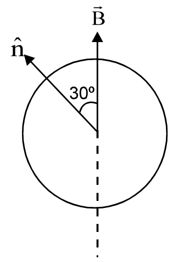
8. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wave length 975A°. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength among them. You may assume the ionization energy of hydrogen atom as 13.6 eV.

9. Average life time of a hydrogen atom excited to n = 2 state is 10^{-8} s. Find the number of revolutions made by the electrons on the average before it jumps to ground state.

10. In a hydrogen like ionized atom a single electron is orbiting around a stationary positive charge. If a spectral line of λ equal to 4861Å is observed due to transition from n = 12 to n = 6. What is the wavelength of a spectral line due to transition from n = 9 to n = 6 and also identify the element.

11. For atoms of light and heavy hydrogen (H and D) find the difference;
(a) between the binding energies of their electrons in the ground state.
(b) between the wavelengths of first lines of the Lyman series.

12. An electron in the ground state of hydrogen atoms is revolving in anti clock wise direction in a circular orbit of radius R. **[JEE 1996, 5]**



(i) Obtain an expression for the orbital magnetic dipole moment of the electron.
(ii) The atom is placed in a uniform magnetic induction, such that the plane normal to the electron orbit make an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.

13. A proton and an electron, both at rest initially, combine to form a hydrogen atom in ground state. A single photon is emitted in this process. What is the wavelength ?

14. A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90º with respect of its original direction. **[JEE 1993; 9 + 1M]**
(a) Find the allowed values of the energy of the neutron and that of the atom after the collision.
(b) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation. [Given : Mass of He atom = 4 × (mass of neutrons Ionization energy of H atom = 13.6 eV]

15. Suppose the potential energy between electron and proton at a distance r is given by $U = ke\,\ell n\frac{r}{a}$, where r < a and k, e, a are positive constants. Use Bohr's theory to obtain the energy of nth energy level for such an atom.

16. A positronium consists of an electron and a positron revolving about their common centre of mass. Derive and calculate
(i) Separation between the electron and positron in their first excited state.
(ii) Kinetic energy of the electron in ground state.

17. In a photo electric effect set – up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV & of radius 8.0×10^{-3}m. The efficiency of photo electrons emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photo electrons are instantly swept away after emission. **[JEE 1995, 10]**
(a) Calculate the number of photo electrons emitted per second.
(b) Find the ratio of the wavelength of incident light to the De – Broglie wave length of the fastest photo electrons emitted.
(c) It is observed that the photo electron emission stops at a certain time t after the light source is switched on. Why?
(d) Evaluate the time t.

18. The K_β x-ray of argon has a wavelength of 0.36 nm . The minimum energy required to take out the outermost electron from argon atom is 16.53 eV. Find the energy (in KeV) needed to knock out an electrons from the K shell of an argon.

19. A schwarzschild black hole is characterized by its mass M and a mathematical spherical surface of radius $R_s = \frac{2GM}{C^2}$ called the event horizon. If the radial distance of an object r from the black hole is such that $r < R_s$, then the object is "swallowed" by the black hole and r rapidly decreased to the singular point r = 0 **[Olympiad_2015]**
(a) Suppose a black hole of mass M "captures" a proton to form a "black hole proton atom (BHP)" in circular orbit. Find the smallest radius r_B of this atom.
(b) Obtain a numerical upper bound on M such that a stable BHP may exist.
(c) Find the minimum energy F_{min}, in Mev, reuired to dissociate this BHP atom from the ground state.
(d) In 1974, Stephen Hawking showed that quantum effects cause black to radiate like a black body with temperature $T_{BH} = \frac{10^{23}K}{M}$. Discuss then the possibility of the existence of a stable BHP atom.

HLP Answers

2. $E = \left(\frac{nh}{2\pi}\right)^6 \frac{1}{6\,(Ke^2)^2\,m^3}$

3. $\frac{13.6}{(4)^2}$ =0.85eV(n = 4 to n= 2)

4. (a) $\frac{9}{8R}$ = 103 nm (b) $\frac{16}{3R}$ = 487 nm

5. $\sqrt{\frac{2}{m_e}.\left[\frac{10^9 hc}{18} - 54.4\ e\right]}$ = 2.2 × 10^6 m/s

6. (I) 913Å, (II) 10.2 eV

7. (a)$\lambda = \frac{hc}{(13.6+1.9)\ eV}$ = 80 nm;(b) $\lambda = \frac{hc}{(10.2+1.9)\ eV}$ = 102 nm ; (c) $\frac{hc}{(12.08+1.9)\ eV}$ = 89 nm

8. 6, $\lambda_{min} = \frac{16\times 9}{7R}$ = 18800 A°,

9. $10^{-8} \times \frac{2.19\times 10^6}{2\pi(0.529\times 10^{-10})} \times \frac{(1)^2}{(2)^3}$ = 8.2 × 10^6

10. 6563 Å, Z = 3

11. $E_D - E_H$ = 3.7 meV, $\lambda_H - \lambda_D$ = 33 pm

12. (i) $\frac{he}{4\pi m}$ (ii) $\frac{h\,e\,B}{8\,\pi\,m}$

13. 912 Å

14. (a) 6.36 eV, 0.312 eV (of neutron), 17.84 eV, 16.328 eV (of atom)
(b) 1.82 × 10^{15} Hz, 11.67 × 10^{15} Hz, 9.84 × 10^{15} Hz.

15. $\frac{1}{2}ke\left(1+2\ln\left(\frac{nh}{2\pi\sqrt{kema^2}}\right)\right)$

16. (i) $r_0 = \frac{2h^2}{\pi^2 m} \times \frac{4\pi\varepsilon_0}{e^2}$ = 4.23 Å (ii) $\frac{1}{2}m\left(\frac{e^2\pi}{4\pi\varepsilon_0 h}\right)^2$ J = 3.4 eV

17. (a) $10^5\ s^{-1}$ (b) 286.18 (d) $\frac{1000}{9}$sec = 111s

18. $\left[\frac{hc}{0.36\times 10^{-9} e} + 16.53\right]$eV = 4 KeV

19. (a) $r_B = \frac{h^2}{GMm^2}$ (b) M < 2 ×10^{11} Kg. (c) $E_{min.}$ = 55MeV

(d) For M = 10^{11} Kg. $T_{BH} = \frac{10^{23}K}{10^{11}}$ = 10^{12}K At this temperature thermal energies kT_{BH} = 82 MeV. The dissociation energy required is 55 MeV. Thus the BHP is thermally unstable.